Gravity 7

The Terrain Correction

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Potential Fields Geophysics: Terrain Week
Objectives for Terrain Week

- Learn about the terrain correction
- The inner terrain correction
- Learn about DEMs
- The outer terrain correction
- Make a terrain correction to gravity data
Background on terrain correction

Terrain corrections deal with the deviation of actual topography from the topography approximated by the Bouguer slab, or spherical Bouguer cap. Consider this highly simplified diagram:

A gravity station is located at height \( h \), corresponding to the Bouguer slab (shaded area in panel \( b \)). However, the valleys (area \( A \) in panel \( c \)) are overcompensated by the Bouguer slab. That is, the mass of the Bouguer slab includes the area in the valley where the mass is absent. Similarly, the mass at the top of the hill (area \( B \) in panel \( c \)) pulls up on the gravity meter but is not accounted for by the simple Bouguer correction. The terrain correction is designed to account for the effects of “real world” topography.

Effect of topography

Note that the effect of valleys is that the simple Bouguer slab correction has removed the effect of mass that was not there to begin with. Therefore after the terrain correction is applied, the terrain corrected gravity should be greater than the simple Bouguer gravity. Likewise, the hilltop, unaccounted for in the simple Bouguer correction, pulls up on the gravity meter. Therefore, like the valley, after the terrain correction is applied, the terrain corrected gravity due to the hilltop should be greater than the simple Bouguer gravity.
Background on terrain correction

Apparent the terrain correction was first considered by Hayford and Bowie, around 1912, in interpreting gravity anomalies in the US. The problem of how to estimate the terrain correction was tackled by the geodesists Cassinis, Bullard, and Lambert in the 1930s. Hammer developed a practical approach for performing terrain corrections out to about 22 km from the station.

Bullard (1936) broke the topographic correction into three parts. The first two are the Bouguer correction (Bullard A), which approximates the topography to an infinite horizontal slab of thickness equal to the height of the station above the reference ellipsoid or another datum plane, and the curvature of the Earth (Bullard B) or spherical cap, which reduces the infinite Bouguer slab a spherical cap of the same thickness, with a surface radius of 166.735 km – 1.5 degree. We have already discussed these corrections in Module 5.

The third correction (Bullard C) is the terrain correction which takes undulations of topography into account. Topographic variations results in the upwards attraction of hills above the plane of the station and valleys below, which decrease the observed value of gravity, so both of these effects must be added to readings to correct for topography.

The complex topography of the Medicine Lake (CA) highlands, with its lava domes, cones, and faults, illustrates the potential significance of deviation from a simple Bouguer slab model of topography.
The Hammer Net

Hammer improved on the method of Hayford to simplify terrain corrections. His “Hammer net” was used for 70 yr to make terrain corrections. The method involves compartmentalizing the area surrounding the measurement point using a template – the Hammer net. The net is used with printed topographic maps. Radial lines from the gravity station extend from the center of the net, and concentric circles drawn at specific distances from the gravity station, consistent with the scale of the topographic map. An average elevation above or below the station elevation within each compartment is estimated.

\[ g_{\text{comp}} = G \rho \Delta \theta \left[ R_o - R_i + \sqrt{R_i^2 + h^2} - \sqrt{R_o^2 + h^2} \right] \]

where: \( \rho \) is the bulk density of the terrain used in the simple or spherical cap Bouguer correction, \( h \) is the height difference of the compartment, \( \Delta \theta \) is the angle subtended by the two radial lines bounding the segment, \( R_i \) and \( R_o \) are the outer and inner radii bounding the compartment. This is the gravity effect of a radial segment of a hollow vertical cylinder, with a flat top (the average elevation difference with the station, \( h \)). The gravitational effect of each compartment is then summed to estimate the terrain correction:

\[ g_{\text{ter}} = \sum_{k=1}^{N} g_{\text{comp}} \]

where: \( N \) is the total number of compartments.

Although Hammer nets are no longer used, generally, this method illustrates the same basic concept still used in computer programs to estimate the terrain correction. That is, elevation data are found as a function of distance and direction from each gravity station, and the gravitational effect of the elevation difference is calculated as a function of distance.
A Hammer net, or similar net used by a computer program looks like this:

For each terrain correction, the net is centered on the gravity station (red dot) and the elevation of the station, $h_s$ is determined, using the terrain model. The elevation difference with each compartment is then estimated. A simple scheme is:

$$h_i' = h_s - h_i$$

$$h_o' = h_s - h_o$$

$$h = \frac{h_i' + h_o'}{2}$$

Note that the innermost zone is a special case, because $h_s = h_i$ and because the correction is most sensitive to topographic variations in this inner zone. Consequently a different inner zone correction is normally used and the Hammer net-like corrections are referred to a outer zone terrain corrections. Usually the inner zone is of 50 m radius, or so.
A simplified example of the Hammer Net

Example

What is the terrain effect of a lava dome range of average relief 500 m located 2–5 km from the gravity meter, $S$? Assume the dome is contained in a compartment defined by $\Delta \theta = 45^\circ$.

\[ g_{comp} = G \rho \Delta \theta \left( R_o - R_i + \sqrt{R_i^2 + h^2} - \sqrt{R_o^2 + h^2} \right) \]

with $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, $\rho = 2300 \text{ kg m}^{-3}$, $\Delta \theta = \pi/4$, $R_i = 2000 \text{ m}$, $R_o = 5000 \text{ m}$:

$g_{comp} = 0.44 \text{ mGal}$

First, realize that this is a substantial effect. If a gravity map were made across the region, the simple Bouguer anomaly would have a gravity gradient, increasing away from the volcano. Also, since topographic features do not occur in isolation, the cumulative effect of topography may be much larger. Second, note that, although this calculation is quite coarse compared to an actual terrain correction, it is possible in the field to quickly and roughly estimate the expected gravitational effect of topographic features with a topographic map and compass.
The inner zone terrain correction is often done by considering the average slope of topography, as a function of direction, near the gravity meter. The gravity anomaly due to the topographic wedge at a gravity station located at the origin (point $S$, red circle, in the figure) is:

$$g_{comp} = \frac{\pi}{2} G \rho R (1 - \cos \theta)$$

where the wedge spans $90^\circ$ azimuth (hence the $\pi/2$ term) and $\rho$ is the density used in the topographic correction, $R$ is the radius of the wedge and $\theta$ is the average slope of the surface.

**Example**

Suppose a Brunton compass is used to estimate the average slope in four directions (at $90^\circ$ azimuths) around a gravity station and the average slopes out to 53 m are $+10^\circ$, $+15^\circ$, $-10^\circ$, and $-5^\circ$. Prove to yourself that the inner zone terrain correction is 2.51 mGal, for a density of 2300 kg m$^{-3}$.

In hilly terrain the inner zone terrain correction can be quite large. In such areas, it may be necessary to measure slopes (or change in elevation with distance) with $R < 50$ m, then use:

$$g_{comp} = G \rho \Delta \theta \left[ R_o - R_i + \sqrt{R_i^2 + h^2} - \sqrt{R_o^2 + h^2} \right]$$

to estimate topographic effects in additional zones, before the DEM is used for more distal zones. Various tools are used to do inner and “nearly” inner zone corrections, including a compass that can measure slope, a laser distance measurement tool, an automatic level or theodolite. Terrestrial LiDAR scanners can also be used for inner zone corrections with very high precision.
In practice, the outer zone terrain correction is made using a digital elevation model (DEM). This is a DEM of the Medicine Lake volcano area (California):

Steps in getting and using a DEM include:

- Download the DEM data. Globally, DEM data are available at 90-m-resolution derived from shuttle radar topography mission (SRTM) data. In many countries, such as the US, 30-m or 10-m DEM data are readily available.

- Most outer terrain correction codes use a UTM Grid (in meters) rather than latitude / longitude coordinate system (in degrees). So it is necessary to convert the DEM, usually, from lat/long to UTM. Program such as Proj4, ArcGIS, or gdal can be used to do this conversion.

- The terrain model must be checked, so that there are no “holes” in the DEM or missing values. These missing values can cause significant errors in terrain corrections using DEMs.

- Usually the data are re-gridded to get regularly spaced rows and columns of elevation values over the region of interest.
The book by Hinze et al. (2013) has an excellent overview of the terrain correction. Various authors have worked on the terrain correction, proposing various inner zone and outer zone corrections. Important papers on the topic include:

1. Suppose you set up the gravity meter to take a measurement next to a curb, 0.2 m in relief. Assume the gravity meter center is 0.2 m from the actual curb edge. Use the inner zone formula to estimate the terrain effect of the curb. Note: assume the curb is straight, so that two 90-degree wedges of the inner zone are above the curb, and two 90-degree wedges are below the curb. Explain your result, including the density value you assume.

2. Suppose you set up a gravity station and you use a laser rangefinder to estimate the vertical height difference between the meter and the ground surface at a horizontal distance of 53 m from the gravity meter in four cardinal directions. These height differences are: 10 m, -5 m, -4 m and 8 m. What is the inner zone terrain correction? Explain your result, including the density value you assume.

3. Suppose you set up a gravity station approximately 30 km from Mt. Shasta volcano. The relief of Mt. Shasta is approximately 3000 m and the diameter approximately 20 km. Estimate the terrain effect of Mt. Shasta on your gravity reading. Explain your result, including the density value you assume.
The outer zone correction can be estimated using the PERL script: `terrain_corr2.pl`. This code uses the “Hammer” formula (hollow right vertical cylinder) to estimate the terrain correction using a configuration file and a digital elevation model. The configuration file specifies the information needed to run the code. This includes:

1. The name of the file containing the gravity data to be corrected. In this case, use the file: “gravity_data_62E_...utm”. This file lists Medicine Lake area gravity values by utm coordinate, simple Bouguer gravity reading, and elevation.
2. The name of the digital elevation model (DEM) file to be used to make the terrain correction.
3. The angular frequency, $\Delta \theta$ with which terrain corrections are made.
4. The thickness of the hollow cylinders used in the terrain correction (distance between $R_o$ and $R_i$).
5. The “skip distance”, that is the distance from the gravity meter to the first value of $R_i$. Terrain in this zone is not corrected for as this is defined as the inner zone.
6. The limits of the DEM

The values shown in the configuration file can be edited to change the terrain correction. Use the 90-m and 30-m DEM files provided to estimate the terrain correction for the Medicine Lake gravity stations. How does the terrain correction change for the gravity stations with change in the resolution of the DEM (from 30 to 90 m)? Using the 30 m DEM, how does the terrain correction change with the angular frequency, $\Delta \theta$, and with the distance between $R_o$ and $R_i$? Summarize your results graphically and explain them!