ATMOSPHERIC AND PLUME DIFFUSION

(from *Bonadonna et al.* [2005])

Considering $f_{i,j}(x,y)$ (m^{-2}) as a function that uses the advection-diffusion equation to estimate the fraction of mass of a given particle size and release height to fall around the point with coordinates (*x,y*), the analytical solution of the mass-conservation equation can be written as:

$$f_{i,j}(x,y) = \frac{1}{2\pi\sigma_{i,j}^2}\exp\left\{-\frac{\left(x-\overline{x}_{i,j}\right)^2 + \left(y-\overline{y}_{i,j}\right)^2}{2\sigma_{i,j}^2}\right\}, \tag{1}$$

where $\overline{x}_{i,j}$ and $\overline{y}_{i,j}$ are the coordinates of the center of the bivariate Gaussian distribution ($\overline{x}_{i,j} = x_i + \sum\limits_{layers}\delta x_j$, $\overline{y}_{i,j} = y_i + \sum\limits_{layers}\delta y_j$) and $\sigma_{i,j}^2$ is the variance of the Gaussian distribution, which is controlled by atmospheric diffusion and horizontal spreading of the plume [*Suzuki*, 1983].

Effectively, the use of $\sigma_{i,j}^2$ in eq. 1 lumps complex plume and atmospheric processes into a single parameter. This greatly simplifies the model, making it much easier to implement but also ignores processes that can affect tephra dispersion. For example, the diffusion coefficient is likely scale dependent and varies with barometric pressure in the atmosphere [e.g. *Hanna et al.*, 1982]. Such factors are not considered in the model.

Atmospheric turbulence is a second order effect for coarse particles, and several models for tephra dispersal are based on the assumption that the atmospheric turbulence is negligible [e.g. *Bonadonna et al.*, 1998; *Bursik et al.*, 1992b; *Sparks et al.*, 1992]. However, if the fall time of particles is large, for example for ash-sized particles, atmospheric turbulence may not be negligible [*Bursik et al.*, 1992a; *Suzuki*, 1983]. For small particle-fall times, $t_{i,j}$, the diffusion is linear (Fick's law), and the variance $\sigma_{i,j}^2$ is [*Suzuki*, 1983]:

$$\sigma_{i,j}^2 = 2K\left(t_{i,j} + t_j'\right) \tag{2}$$

where K (m^2 s^{-1}) is a constant diffusion coefficient and t_i' (s) is the horizontal diffusion time in the vertical plume. The horizontal diffusion coefficient, K, is considered isotropic ($K=K_x=K_y$) [*Armienti et al.*, 1988; *Bonadonna et al.*, 2002; *Connor et al.*, 2001; *Hurst and Turner*, 1999; *Suzuki*, 1983]. The vertical diffusion coefficient is small above 500 m of altitude [*Pasquill*, 1974], and therefore is assumed to be negligible.

The horizontal diffusion time, $t_i^{'}$, accounts for the change in width of the vertical plume as a function of height, which is a very complex process [*Ernst et al.*, 1996; *Woods*, 1995]. Such a change in plume width simply adds to the dispersion of tephra fall, and so can be expressed as $t_i^{'}$ [*Suzuki*, 1983]. Here, we approximate the radius, r_i, of the spreading plume at a given height, z_i, with the relation developed by *Bonadonna and Phillips* [2003] and based on the combination of numerical studies [*Morton et al.*, 1956] and observations of plume expansion [*Sparks and Wilson*, 1982]: $r_i = 0.34 z_i$. Thus, taking $r_i = 3\sigma_p = 3\sigma_{i,j}$, where σ_p is the standard deviation of the Gaussian distribution of the mass in the ascending plume [*Sparks et al.*, 1997; *Suzuki*, 1983], and from eq. 2 with $t_{i,j} = 0$ we have:

$$t_i^{'} = \frac{0.0032 z_i^2}{K} \; .$$

(3)

When the particle fall time is of a scale of hours, the scale of turbulent structures that carry particles increases with time [*Suzuki*, 1983]. As an example, particles with diameter <1 mm falling from a 30 km-high plume will have an average fall time >1 hour (based on their particle settling velocity). In this case the variance $\sigma_{i,j}^2$ can be empirically determined as [*Suzuki*, 1983]:

$$\sigma_{i,j}^2 = \frac{4C}{5} \left(t_{i,j} + t_j^{'} \right)^{2.5}$$

(4)

where C is the apparent eddy diffusivity determined empirically ($C = 0.04$ m^2 s^{-1}; [*Suzuki*, 1983]). Taking $t_{i,j} = 0$ in eq. 4 and $r_i = 3\sigma_{i,j} = 0.34 z_i$ as for eq. 3, we have that the horizontal diffusion time for fine particles is:

$$t_i^{'} = \left(0.2 z_i^2 \right)^{\frac{2}{5}} .$$

(5)

Figures 1a and 1c show how $t_i^{'}$ significantly affects the total fall time of *coarse* particles more than the total fall time of *fine* particles, i.e. $(t_{i,j} + t_i^{'})$, because for *fine* particles $t_i^{'} << t_{i,j}$ (Figs 1a and 1b). However, depending on the value of K, the horizontal diffusion time of *coarse* particles is typically smaller than the horizontal diffusion time of *fine* particles for low heights (Fig. 1b). In this case *coarse* and *fine* particles indicate particles with fall time < or > than the fall-time threshold chosen as a transition between eq. 2 and eq. 4 (e.g. fall-time threshold = 3600 s in Fig. 1). Such a transition is not well defined based on theory but can be determined empirically.

As a conclusion, once particles leave the bottom of the turbulent current, they experience different types of turbulent diffusion depending on their size. The linear diffusion

described by eq. 2 strongly depends on the choice of the diffusion coefficient, whereas the power-law diffusion described by eq. 4 strongly depends on the particle fall time and the horizontal diffusion time of the ascending plume [*Suzuki*, 1983]. If the volcanic plume is sufficiently high, some particles will experience a shift in diffusion law during fall due to the decrease in fall time (e.g. particles with diameter of 0.25 mm in Fig. 1d). Figure 1d also shows the strong power-law dependence of $\sigma_{i,j}^2$ on time, which makes the total diffusion more significant for *fine* particles.
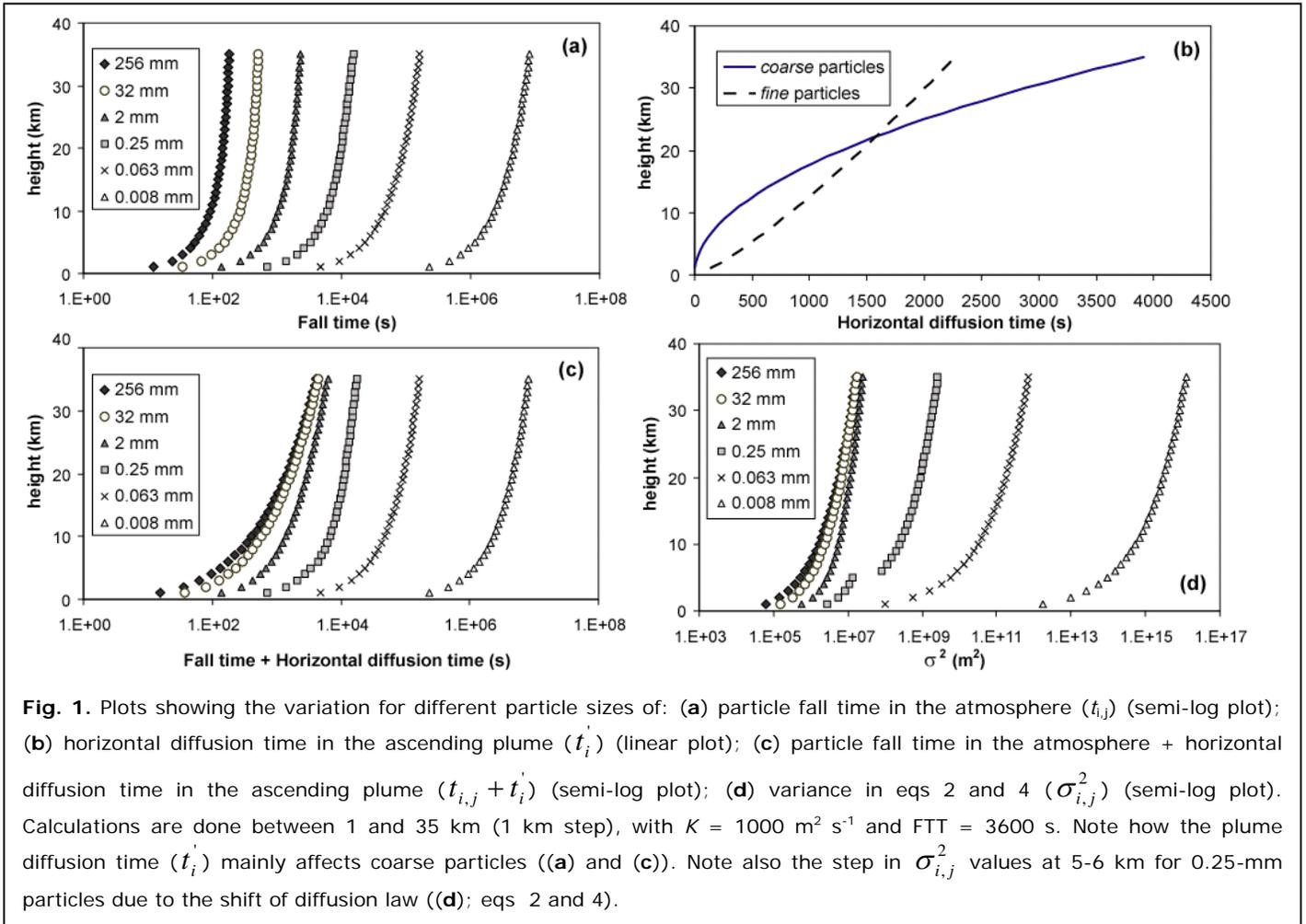

Fig. 1. Plots showing the variation for different particle sizes of: (**a**) particle fall time in the atmosphere ($t_{i,j}$) (semi-log plot); (**b**) horizontal diffusion time in the ascending plume (t_i') (linear plot); (**c**) particle fall time in the atmosphere + horizontal diffusion time in the ascending plume ($t_{i,j} + t_i'$) (semi-log plot); (**d**) variance in eqs 2 and 4 ($\sigma_{i,j}^2$) (semi-log plot). Calculations are done between 1 and 35 km (1 km step), with K = 1000 m^2 s^{-1} and FTT = 3600 s. Note how the plume diffusion time (t_i') mainly affects coarse particles ((**a**) and (**c**)). Note also the step in $\sigma_{i,j}^2$ values at 5-6 km for 0.25-mm particles due to the shift of diffusion law ((**d**); eqs 2 and 4).

References

Armienti, P., G. Macedonio, and M.T. Pareschi, A numerical-model for simulation of tephra transport and deposition - applications to May 18, 1980, Mount-St-Helens eruption, *Journal of Geophysical Research*, *93* (B6), 6463-6476, 1988.

Bonadonna, C., C.B. Connor, B.F. Houghton, L. Connor, M. Byrne, A. Laing, & T. Hincks. Probabilistic modeling of tephra dispersion: hazard assessment of a multi-phase eruption at Tarawera, New Zealand. *Journal of Geophysical Research, 110*, 2005.

Bonadonna, C., G.G.J. Ernst, and R.S.J. Sparks, Thickness variations and volume estimates of tephra fall deposits: the importance of particle Reynolds number, *Journal of Volcanology and Geothermal Research*, *81* (3-4), 173-187, 1998.

Bonadonna, C., G. Macedonio, and R.S.J. Sparks, Numerical modelling of tephra fallout associated with dome collapses and Vulcanian explosions: application to hazard assessment on Montserrat, in *The eruption of Soufrière Hills Volcano, Montserrat, from 1995 to 1999*, edited by T.H. Druitt, and B.P. Kokelaar, pp. 517-537, Geological Society, London, Memoir, 2002.

Bonadonna, C., and J.C. Phillips, Sedimentation from strong volcanic plumes, *Journal of Geophysical Research*, *108* (B7), 2340-2368, 2003.

Bursik, M.I., S.N. Carey, and R.S.J. Sparks, A gravity current model for the May 18, 1980 Mount-St-Helens plume, *Geophysical Research Letters*, *19* (16), 1663-1666, 1992a.

Bursik, M.I., R.S.J. Sparks, J.S. Gilbert, and S.N. Carey, Sedimentation of tephra by volcanic plumes: I. Theory and its comparison with a study of the Fogo A plinian deposit, Sao Miguel (Azores), *Bulletin of Volcanology*, *54*, 329-344, 1992b.

Connor, C.B., B.E. Hill, B. Winfrey, N.M. Franklin, and P.C. La Femina, Estimation of volcanic hazards from tephra fallout, *Natural Hazards Review*, *2* (1), 33-42, 2001.

Hanna, S.R., G.A. Briggs, and R.P. Hosker, *Handbook on atmospheric diffusion*, 102 pp., Technical Information Center (U.S. Department of Energy), 1982.

Hurst, A.W., and R. Turner, Performance of the program ASHFALL for forecasting ashfall during the 1995 and 1996 eruptions of Ruapehu volcano, *New Zealand Journal of Geology and Geophysics*, *42* (4), 615-622, 1999.

Morton, B., G.L. Taylor, and J.S. Turner, Turbulent gravitational convection from maintained and instantaneous source, *Proceedings of the Royal Society*, *234*, 1-23, 1956.

Sparks, R.S.J., M.I. Bursik, G.J. Ablay, R.M.E. Thomas, and S.N. Carey, Sedimentation of tephra by volcanic plumes .2. Controls on thickness and grain-size variations of tephra fall deposits, *Bulletin of Volcanology*, *54* (8), 685-695, 1992.

Sparks, R.S.J., M.I. Bursik, S.N. Carey, J.S. Gilbert, L.S. Glaze, H. Sigurdsson, and A.W. Woods, *Volcanic Plumes*, 574 pp., John Wiley & Sons, Chichester, 1997.

Sparks, R.S.J., and L. Wilson, Explosive volcanic eruptions: V. Observations of plume dynamics during the 1979 Soufrière eruption, St. Vincent, *Geophysical Journal of the Royal Astronomical Society*, *69*, 551-570, 1982.

Suzuki, T., A theoretical model for dispersion of tephra, in *Arc Volcanism, Physics and Tectonics*, edited by D. Shimozuru, and I. Yokoyama, pp. 95-113, Terra Scientific Publishing Company (TERRAPUB), Tokyo, 1983.